THE COLLEGES OF OXFORD UNIVERSITY

Mathematics

15 December 1996

Time allowed: $2\frac{1}{2}$ hours

For candidates applying for Mathematics, and Joint Schools with Mathematics.

Write your name and College of preference below in BLOCK CAPITALS.

Name: **College of preference:**

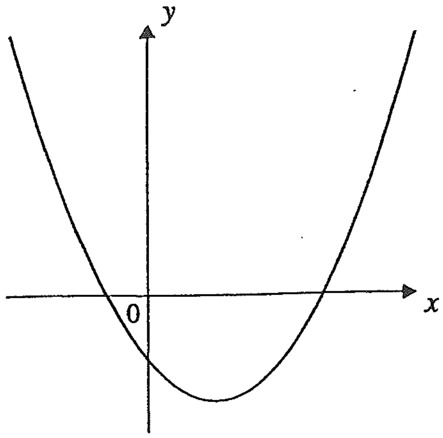
Answer all questions.

Place your answers to Question 1 in the table below. Write your answers to Questions 2 to 5 in the space provided. Additional sheets of paper may be inserted.

1. Place a tick ($\surd$) in the appropriate box.

Question 1	Answer			
Part	(i)	(ii)	(iii)	(iv)
(a)				
(b)				
(c)				
(d)				
(e)				
(f)				
(g)				
(h)				
(j)				
(k)				

For each part choose the correct answer from (i) - (iv). There is only one in each case.



(a) The diagram above shows the graph of the function $y = ax^2 + bx + c$. Then:

(i) $b^2 - 4ac > 0$; (ii) $b^2 - 4ac = 0$; (iii) $b^2 - 4ac \leq 0$; (iv) $b^2 - 4ac < 0$.

(b) The inequality $2^n > n^2$ is true for:

(i) no integers $n \geq 0$; (ii) all integers $n \geq 0$;

(iii) all integers $n > 4$; (iv) all integers $n \geq 4$.

(c) The simultaneous equations

$$ax + by = 1$$
$$cx + dy = 0$$

in x and y:

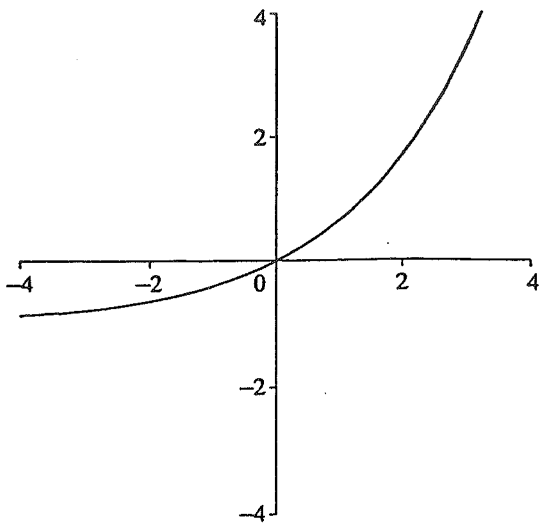
(i) have a solution whatever the values of a, b, c, d may be;
(ii) have a unique solution whatever the values of a, b, c, d may be;
(iii) have a solution only if $ad \neq bc$;
(iv) have a unique solution only if $ad \neq bc$.

(d) The complete set of solutions of the equation $\sin 2x = \cos x$ in the range $0 \leq x \leq 2\pi$ is:

(i) $\{\pi/2, 3\pi/2\}$; (ii) $\{\pi/6, 5\pi/6\}$; (iii) $\{\pi/6, \pi/2\}$; (iv) $\{\pi/6, \pi/2, 5\pi/6, 3\pi/2\}$.

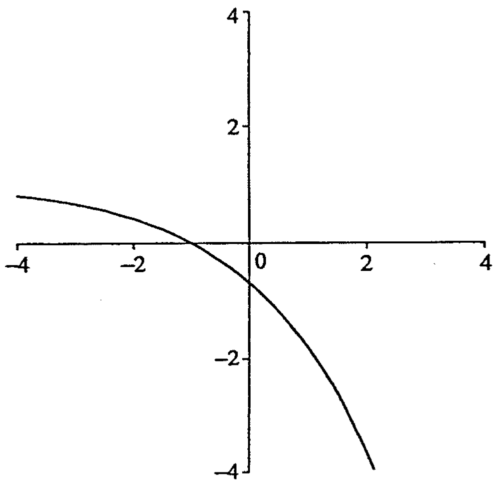
(e) If $|x - 3| < 1$ and $|x - 1| > 2$, then:

(i) $-1 < x < 4$; (ii) $3 < x < 4$; (iii) $2 < x < 3$; (iv) $2 < x < 4$.

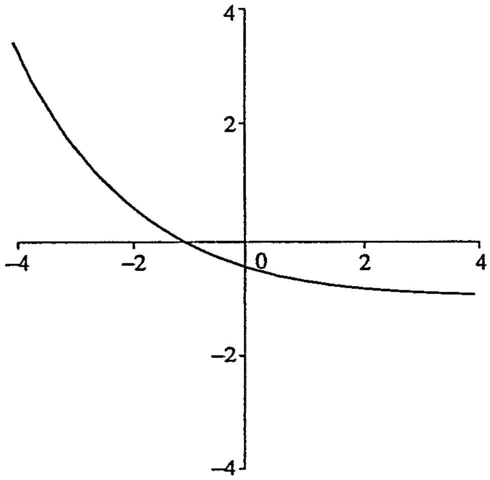


(f) The diagram above shows the graph of the function $y = f(x)$. The graph of the function $y = -f(x+1)$ is:

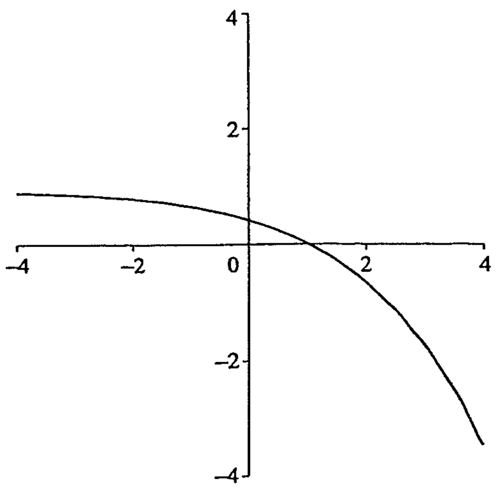
(i)



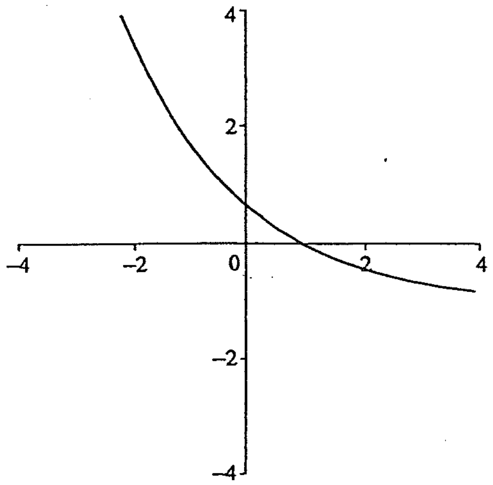
(ii)



(iii)



(iv)



(g) As n becomes very large and positive, $10000^{-\frac{1}{n}}$ approaches:

(i) 0; (ii) 1; (iii) 10000; (iv) ∞.

(h) The derivative of the function $y = (e^{\cos(5x)})^2$ is:

(i) $-5\sin(5x)(e^{\cos(5x)})^2$; (ii) $-20\sin(5x)\cos(5x)(e^{\cos(5x)})^2$;

(iii) $-10\sin(5x)(e^{\cos(5x)})^2$; (iv) $-10\sin(5x)\cos(5x)(e^{\cos(5x)})^2$.

(j) The derivative of the function

$$F(x) = \int_0^x f(t)\,dt$$

is:

(i) $f(x) - f(0)$; (ii) $f'(x)$; (iii) $f(x)$; (iv) $f'(x) - f'(0)$.

(k) An entrance candidate is dealt three cards from a pack of fifty-two playing cards. To one significant figure the probability that he receives exactly one king is:

(i) 0.003; (ii) 0.01; (iii) 0.2; (iv) 0.05.

[There are four kings in a pack of playing cards.]

2. (a) Factorise the expression $x^2 + x - 6$.

(b) For which values of the real constant a does the equation

$$x^2 + x - a = 0$$

have at least one real solution? Write down these solutions in terms of a.

(c) Show that, for any value of the real constant b, the equation

$$x^3 - (b+1)x + b = 0$$

has $x = 1$ as a solution. Find all values of b for which this equation has exactly two distinct solutions.

3. (a) Write down the equation of the straight line through the point $(1, 2)$ with slope -1.

(b) Let l be a line with equation

$$y = (2 - a) + ax,$$

where a is a constant. Show that, for any a, the line passes through the point $(1, 2)$. Find the equation of the line perpendicular to this line which also passes through the point $(1, 2)$.

(c) Find the equations of the lines which pass through the point $(1, 2)$ and have perpendicular distance 1 from the origin.

4. (a) Find the values of

(i) $\displaystyle\int_{-1}^{1} (x^2 - x)\,\mathrm{d}x$,

(ii) $\displaystyle\int_{-1}^{1} (x^3 + x^2 - 2x)\,\mathrm{d}x$.

(b) Sketch the graph of $y = x^2 - x$ and indicate which difference in areas is represented by your answer to (a)(i).

(c) Find the total area (measured positively) that lies between the graphs of $y = x^2 - x$ and $y = x^3 + x^2 - 2x$ between $x = -1$ and $x = 1$.

(d) The answers to (a)(i) and (a)(ii) are related in a particular way. Explain how the relationship can be seen *without* working out any integrals.

5. A total of 12 noughts and 4 crosses are arranged in 4 rows of 4. One such arrangement is illustrated below.

0	0	×	0
0	×	0	×
0	0	0	0
×	0	0	0

(a) How many arrangements are there altogether?

(b) How many arrangements are there in which there is a cross in every row?

(c) How many arrangements are there in which there is a cross in every row and in every column?